A Strategy for STEM Learning in a Changing Society: Focusing on the Undergraduate Program

Mathematics Education Using Real-World Problems

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Background

- It is increasingly important for all students at all educational levels to acquire mathematical skills for use in real-world situations.

- An important issue in Japan: mathematical literacy education for ‘Bunkei’ students (=humanities & social sciences students)

  - Aims of mathematical literacy educations for Bunkei students:
    - to foster students’ mathematical skill for use in real-world situations
    - to improve students’ attitude towards math
  
- However, it is challenging because many of those students have
  - math anxiety and difficulty in learning math,
  - little knowledge about how mathematics is used in the real world.

- **Question:** How should we design mathematical literacy education for Bunkei students?
New Math Courses (2012-)

- Math courses for the humanities & social sciences students of the first year at Osaka Prefecture University.
  - Basic Math I (spring semester)
  - Basic Math II (Autumn semester)
  - 90 min/week for 14 weeks followed by an examination period
  - 4 classes taught by 4 teachers (60-80 students in each class)
Design Principles

(Kawazoe et al., 2013; Kawazoe & Okamoto, 2017)

1. Design lessons according to *mathematical modelling processes*.
2. Choose *topics & contexts* by considering which mathematical knowledge students are likely to *encounter in real life* and in which *situation* they will encounter it.
3. *Present problems in different contexts* associated with the same mathematical knowledge.
4. *Connect different mathematical knowledge together* by using different mathematizations of the same problem or mutually related contexts.
5. Explain mathematical concepts & operations using *both mathematical language & everyday language*.
6. Engage students in *group activities* rather than individual ones.
7. Design worksheets based on analysis of *students’ understanding process* and use them as *tools for formative assessment*. 
Design Principles

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1. Design lessons according to mathematical modelling processes.

- **Problem in Real World**
- **Discussion**
- **Prediction**
- **Mathematization**
- **Mathematical Model**
- **Validation**
- **Working Mathematically**
- **Answer for Real-world Problem**
- **Mathematical Result**
- **Interpretation**
- **Reflection**

START!
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Basic Math I (Spring semester)
- systems of linear equations/inequalities (linear programming)
- number sequences (savings, loan payment, pharmacokinetics, model of addiction, etc.)
- matrices & vectors (spreadsheets, social networks analysis)
- functions (mental rotation, pharmacokinetics, bacterial growth, pandemic, etc.)
- probability (lottery, disease examination, birthday paradox, Bayesian estimation)

Basic Math II (Autumn semester)
- eigenvalues/vectors (population dynamics, optimal distribution, etc.)
- functions (cyclical movement of electric demand, sound composition, etc.)
- derivatives (innovation diffusion, population growth, logistic function, marginal profit)
- integrals (speed & distance, accumulated radiation level, standard normal distribution)
- multivariable functions (loan simulator, formulas for estimating vital capacity, optimization problem, etc.)
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**Classroom materials (Examples)**

- **Context** Loan payment
- **Context** Blood fexofenadine level (Pharmacy)
- **Context** Blood caffeine level after drinking a cup of coffee
- **Context** Salmonella growth
- **Context** Analyzing the spreading pattern of disease. (FMD outbreak, H1N1)

**Number sequences**
- Recurrence relation: $a_{n+1} = c a_n + d$
- Geometric progression: $a_{n+1} = c a_n$

**Functions**
- Exponential Function
- Semi-Log Plot

**Context**
- Rivo payment of shopping
- Considering a newspaper article with a semi-log graph of radioactive cesium concentration in soil after Fukushima nuclear disaster

**Items in Final Exam (Examples)**
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Example: Eigenvalues/vectors (2-weeks lesson)

**Rental Bicycle Service Problem**

- 2 Parkings (A, B)
- Bicycles can be dropped off at any of the 2 parkings
- Monitoring investigation (weekly data)
- The service will start with more than 100 bicycles.
- How to divide bicycles between 2 parkings?

*Students already learned the transition matrix, but have not learned eigenvectors/values yet.*

<table>
<thead>
<tr>
<th>From \ To</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>70</td>
<td>30</td>
</tr>
<tr>
<td>B</td>
<td>20</td>
<td>80 (%)</td>
</tr>
</tbody>
</table>
1st week: discovering eigenvectors

• How to divide bicycles between the 2 parkings? (*Group discussion*)

**Simulation! What occurs?**

• Choose an initial data and calculate the distribution of the following 5 weeks. (*Group activity*)
  \[
  \begin{pmatrix}
  0.7 & 0.2 \\
  0.3 & 0.8
  \end{pmatrix}
  ^n
  \begin{pmatrix}
  200 \\
  100
  \end{pmatrix}
  =
  \begin{pmatrix}
  ??? \\
  ???
  \end{pmatrix}
  
  From \ To | A | B
  ---|---|---
  A | 70 | 30
  B | 20 | 80 (%)

• Plot the data of all groups on the graph paper. (*Group activity*)

**What can be found?**

• Eigenvectors/values are introduced.
2nd week: explain with eigenvectors

- Re-interpretation:

\[
\begin{pmatrix}
200 \\
100
\end{pmatrix} = \text{??} \begin{pmatrix}
2 \\
3
\end{pmatrix} + \text{??} \begin{pmatrix}
-1 \\
1
\end{pmatrix}
\]

\[
P^n \begin{pmatrix}
200 \\
100
\end{pmatrix} = P^n \begin{pmatrix}
120 \\
180
\end{pmatrix} + P^n \begin{pmatrix}
80 \\
-80
\end{pmatrix}
\]

\[
= \begin{pmatrix}
120 \\
180
\end{pmatrix} + 0.5^n \begin{pmatrix}
80 \\
-80
\end{pmatrix}
\]

- What is an optimal solution?

✓ Optimal solution
  = stable point
  = eigenvector with eigenvalue 1

- Homework:

✓ a similar problem, but in another context
Evaluation of Teaching Practices

• The average of mean scores of the final exam (2012-2018):
  • 72.3 (BMI) & 74.2 (BMII) (full mark=100)

• Results of a self-report questionnaire (Kawazoe et al., 2013):
  • The rates of students who answered that their interest in math increased during the semester were 60.4% (BMI) & 60.6% (BMII).

Results of Self-evaluation (1) (Result of 2012 academic year)

“How did your interest on mathematics change during the semester?”

- Basic Math I (n=285):
  - Not changed: 49%
  - Increased: 35%
  - Decreased: 11%
  - Much increased: 3%
  - Unanswered: 2%

- Basic Math II (n=226):
  - Not changed: 55%
  - Increased: 35%
  - Decreased: 2%
  - Much increased: 6%
  - Unanswered: 2%
• The rates of students who answered that their *mathematical thinking skills improved* during the semester were 74.0% (BMI) & 70.8% (BMII).
• Analysis of students’ free comments in worksheets (BMI in 2018):
  • The frequency of comments containing descriptions showing meta-cognitive reflection per student was greater for students who did not like mathematics than for students who liked mathematics at the beginning of the course. (Kawazoe, 2019)
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Thank you.

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